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Accelerated MRI Thermometry by Direct Estimation of Temperature from Undersampled k-Space Data

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Abstract

**Purpose:** Acceleration of MR thermometry is desirable for several applications of MR-guided focused ultrasound, such as those requiring greater volume coverage, higher spatial resolution, or higher frame rates.

**Theory and Methods:** We propose and validate a constrained reconstruction method that estimates temperature changes directly from k-space without spatial or temporal regularization. A model comprising fully-sampled baseline images is fit to undersampled k-space data, which removes aliased temperature maps from the solution space. Reconstructed temperature maps are compared to maps reconstructed using conventional hybrid thermometry, parallel imaging (SPIRiT), and temporally-regularized thermometry (TCR) techniques.

**Results:** Rapid temperature change simulations demonstrate finer temporal resolution and lower error in $4\times$-undersampled k-space reconstructions compared to TCR. In multi-coil simulations, the k-space method maintains low error using only a few readout lines, showing considerable improvement over single-coil k-space and SPIRiT-reconstructed temperature maps. Phantom heating experiments further demonstrate the algorithm’s advantage over reconstructions relying on parallel imaging alone to overcome undersampling artifacts. In vivo model error comparisons show the algorithm achieves low temperature error at much higher acceleration factors (up to $32\times$) than conventional CG and SPIRiT-based reconstructions.

**Conclusion:** High acceleration factors can be achieved using the proposed temperature reconstruction algorithm, without sacrificing temporal resolution or accuracy.

Key words: temperature imaging; image reconstruction; magnetic resonance imaging
Introduction

Recent technological developments have combined thermal therapies with MRI for targeting, temperature monitoring and assessment of thermal dose in the brain and body [1]. In particular, MRI-guided focused ultrasound (MRgFUS) is increasingly being applied to treat uterine fibroids and cancer [2–5], relieve pain from bone metastases [6, 7], and stimulate deep brain tissue to treat neurological conditions [8, 9]. Additionally, animal studies have demonstrated that MRgFUS can induce gene expression [10] and also facilitate drug delivery [11], even across the blood-brain barrier [12, 13].

In MRgFUS, ultrasound energy is generated outside the body and delivered noninvasively to a specific location within the tissue, ideally without affecting any tissues outside the targeted volume. The primary role of MRI in the procedures is to provide images with soft tissue contrast for targeting, and to provide real-time temperature measurements to monitor thermal dose to the targeted tissue, where temperature measurements are based on the proton resonance frequency shift with temperature. While acoustic energy is nominally focused to a single point in the body, there is an ever-present risk that dangerous heating may occur in other regions in the near- and far-fields of the transducer, and it is therefore necessary to measure temperature rises in real time throughout the tissue to ensure patient safety.

Currently, MRgFUS procedures are limited to 2D temperature imaging of a handful of slices in real-time. A typical thermometry protocol for closed-loop ablation of uterine fibroids uses multishot echo-planar imaging (EPI) to achieve a resolution of $2.5 \times 2.5 \times 7 \text{mm}^3$ in six image slices (three at the focus, one each in the near- and far-fields, and one parallel to the beam path) [14, 15], repeated approximately every three seconds. However, due to its relatively limited volume coverage such a scan can miss dangerous heating that can occur outside the target volume, particularly near tissue interfaces and bones [16]. Therefore, current MR thermometry protocols do not meet the need for complete volume coverage at a high frame rate.

Some form of acceleration is needed to significantly increase the volumetric coverage of MR thermometry without sacrificing frame rate. Current approaches to accelerating MR thermometry
can be loosely grouped into three categories: parallel imaging, compressed sensing, and temporal regularization approaches. Parallel imaging with multiple receive coils is commonly used to accelerate anatomical and functional MR imaging [17, 18], and has been applied to thermometry [19–23]. However, to achieve high accelerations, parallel imaging requires many coils in close proximity with the body, which is often impossible in MRgFUS since the FUS apparatus is large and must be in direct contact with the body. Because of these limitations, the development of MRgFUS-compatible coil arrays is an active area of research, and current state-of-the-art arrays still comprise many fewer coils than are typically used in conventional parallel imaging [24–26].

In addition to parallel imaging, compressed sensing [27] has also been applied to MR thermometry [28, 29]. However, these techniques rely on compressibility of image magnitude to constrain the solution space, and errors can arise when the phase component of the image has high spatial frequency variations such as those caused by heating [30]. Furthermore, compressed sensing must typically be combined with parallel imaging to achieve high accelerations. Compressed sensing reconstructions that incorporate phase regularization have been proposed for thermometry [30, 31], but to date no such method has been described that robustly enables high acceleration factors in a wide variety of acquisition scenarios.

Temporal regularization-based approaches [23, 32–34] assume that images or temperature change slowly during treatment. For example, the temporally constrained reconstruction (TCR) method [32] jointly reconstructs treatment images using a temporal roughness penalty. This enables acceleration when the k-space undersampling pattern is alternated in time so that aliasing artifacts move between consecutive images and are filtered out by the penalty. Other temporal regularization-based approaches have leveraged the bioheat equation and/or Kalman filtering, either to similarly suppress rapidly-changing aliasing artifacts while preserving slower temperature changes, or to interpolate between fully-sampled temperature images acquired at a lower frame rate [33, 34]. While all these approaches are capable of producing temperature maps at any desired frame rate, due to regularization their true temporal resolution lies somewhere between the accelerated and fully-sampled frame rates. Furthermore, the alternating k-space sampling patterns
required by some methods may lead to increased eddy current distortions, and generally limit ac-
quision design.

This work proposes a novel approach to accelerated temperature imaging, that is based on fitting a constrained treatment image model directly to undersampled k-space data. Because the treatment image model comprises fully-sampled pretreatment/baseline images, aliased temperature maps are removed from the solution space, without spatiotemporal regularization. The method is compatible with parallel imaging and can be used with any readout trajectory. Simulations will demonstrate improvements in temporal resolution compared to TCR, and that the method is compatible with and in some cases benefits from parallel imaging. Experiments will investigate in vivo model error as a function of acceleration, and compare the method to conventional parallel imaging reconstructions. Aspects of this work have been previously reported elsewhere [35, 36].

Theory

Signal Model and Problem Formulation

The proposed method estimates temperature maps from undersampled k-space data by fitting a constrained image model directly to the data, without an explicit image reconstruction step. The hybrid multibaseline and referenceless treatment image model is used [37], which comprises a weighted combination of fully-sampled baseline images acquired prior to heating, a polynomial phase shift to model center frequency drift and other bulk phase shifts unrelated to heating, and a spatially-sparse heating-induced phase shift. Sparsity of the heat-induced phase map is exploited by the algorithm to separate it from the bulk phase shifts, and reflects the fact that in a targeted thermal therapy like MRgFUS, temperature rises will occur in a minority of image voxels. When applied to estimate temperature maps from fully-sampled images, hybrid thermometry has been shown to produce accurate temperature measurements in a variety of imaging scenarios, even in the presence of tissue motion [37, 38].
The k-space signal model is the discrete Fourier transform (DFT) of the hybrid image model:

\[ y_i = \sum_{j=1}^{N_s} e^{i\mathbf{k}_i \cdot \mathbf{x}_j} \left( \sum_{l=1}^{N_b} b_{l,j} w_l \right) e^{i \left( \{Ac\}_j + \theta_j \right)} + \varepsilon_i, \]  

where \( y_i \) is one k-space data sample, \( i = 1, \ldots, N_k \) indexes the \( N_k \) acquired samples, \( N_s \) is the number of image voxels, \( \mathbf{k}_i \) is the k-space location of sample \( i \), the \( \{b_l\}_{l=1}^{N_b} \) are complex baseline library images reconstructed from fully-sampled k-space data acquired prior to treatment, the \( w_l \) are baseline image weights, \( A \) is a matrix of smooth (e.g., low-order polynomial) basis functions, \( c \) is a polynomial coefficient vector, \( \theta \) is a heating-induced phase shift, which is negative for a temperature increase [39], and \( \varepsilon \) is complex Gaussian noise. To estimate \( \theta \), the signal model is fit to acquired k-space data \( \tilde{y} \) by solving the constrained minimization problem:

\[
\text{minimize} \quad \frac{1}{2} \| \tilde{y} - y(w, c, \theta) \|^2 + \lambda \| \theta \|_1,
\]

\[ \text{subject to} \quad \theta \leq 0, \]

\[ \sum_{l=1}^{N_b} w_l = 1 \]

\[ w \geq 0, \]

where the first term in the objective function is proportional to the negative log-likelihood of the data (when neglecting noise in the baseline images), \( \| \theta \|_1 \) is the \( \ell_1 \) norm of \( \theta \), and \( \lambda \) is a regularization parameter that controls the sparsity of \( \theta \).

The described method can reconstruct artifact-free temperature maps from undersampled k-space data because the fully-sampled baseline image component of the model removes images (and corresponding temperature maps) from the solution space that contain aliasing artifacts in their magnitude. Solutions containing aliasing artifacts in their phase that are not modeled by the polynomial or sparse heating-induced phase shifts are also eliminated from the solution space.

Algorithm

A solution to the problem in Eq. 2 is found using the following alternating minimization algorithm, given initial estimates of \( w \), \( c \), and \( \theta \):
1: repeat
2: Update w: A quadratic programming problem,
\[
\begin{align*}
\text{minimize} \quad & \frac{1}{2} \left\| \mathbf{y} - \mathbf{G} \text{diag} \left\{ e^{i \left( (\mathbf{A} \mathbf{c}) + \mathbf{\theta} \right)} \right\} \mathbf{B} \mathbf{w} \right\|^2 \\
\text{subject to} \quad & \sum_{l=1}^{N_b} w_l = 1 \\
& \mathbf{w} \succeq 0,
\end{align*}
\]
(3)
is solved, where \( \mathbf{G} \) is a (possibly non-uniform) DFT matrix, and \( \mathbf{B} \) is a matrix whose columns are the baseline images.
3: Update \( \mathbf{\theta} \): The following constrained minimization problem is solved using the nonlinear conjugate gradient (NLCG) algorithm described in the Appendix (J A Fessler, Image reconstruction: Algorithms and Analysis, to be published.):
\[
\begin{align*}
\text{minimize} \quad & \frac{1}{2} \left\| \mathbf{y} - \mathbf{G} \text{diag} \left\{ e^{i \mathbf{\theta}} \right\} \mathbf{f} \right\|^2 - \lambda \sum_{j=1}^{N_s} \theta_j \\
\text{subject to} \quad & \mathbf{\theta} \preceq 0,
\end{align*}
\]
(4)
where \( f_j \triangleq e^{i (\mathbf{A} \mathbf{c})_j} (\mathbf{B} \mathbf{w})_j \).
4: Update \( \mathbf{c} \): The update for \( \mathbf{c} \) is similar to the \( \mathbf{\theta} \) update, except that the gradient, and consequently the NLCG search direction, must incorporate the basis matrix \( \mathbf{A} \),
\[
\nabla_{\mathbf{c}} \Psi = \mathbf{A}' \nabla_{\mathbf{\theta}} \Psi,
\]
(5)
and there are no nonpositivity constraints.
5: until Stopping criterion met
6: To eliminate temperature bias due to the \( \ell_1 \) norm, steps 1-5 are repeated with \( \lambda = 0 \), and \( \mathbf{\theta} \) is only updated in voxel locations that were more negative than a threshold value after Step 5.

Parallel Imaging

When imaging with more than one receive coil, \( (\mathbf{w}, \mathbf{c}, \mathbf{\theta}) \) are simultaneously fit to all coils’ data. Because the baseline images are weighted by the coil sensitivities, the algorithm implicitly performs a SENSE reconstruction [17, 40], without requiring a separate sensitivity map measurement.
It will be demonstrated in simulations that the algorithm is able to exploit multiple receive coils to improve temperature map accuracy at higher accelerations, compared to a single receive coil. Incorporating fully-sampled pre-treatment images provides knowledge about underlying magnitude and phase characteristics of the body. Using this information to constrain undersampled reconstructions guides the algorithm to converge on solutions that do not contain aliasing artifacts. Modeling of smoothly varying background phase and heat-induced phase changes accounts for those deviations from the baseline images that occur during the heating treatment and scanning session. The following experiments and simulations are designed to test the hypothesis that this reconstruction approach will enable temperature map reconstruction from undersampled data without loss of accuracy. In these evaluations, we include comparisons to conventional parallel imaging approaches to accelerate thermometry and to a contemporary, thermometry-specific acceleration approach.¹

Methods

Algorithm Implementation

The proposed k-space algorithm was implemented in MATLAB R2013a (Mathworks, Natick, MA, USA) on a workstation with a 3.4 GHz E31270 Intel Xeon CPU (Intel Corporation, Santa Clara, CA, USA) and 16 GB of RAM. The DFT matrix $G$ was evaluated using non-uniform fast Fourier transforms [41]. Baseline images were reconstructed from fully-sampled k-space data using a conjugate gradient algorithm [42]. The baseline image weights $w$ were solved for using MATLAB’s `quadprog` function. The polynomial phase coefficient vector $c$ and temperature phase shift vector $\theta$ were initialized to zeros. Updates for $c$ and $\theta$ used 5 iterations. Voxels where $\theta$ was less than or equal to -0.01 radians were corrected for temperature bias due to the $\ell_1$ norm as described in Step 6 of the algorithm. In this stage, $c$ and $\theta$ updates contained 5 and 10 iterations, respectively.

¹Example MATLAB files for temperature reconstruction of phantom heating data using the proposed algorithm are available at http://www.vuiis.vanderbilt.edu/~grissowa/.
tively. The algorithm stopped when the relative change in the objective function value was less than 0.1% between consecutive iterations. In the following simulations and experiments, the proposed k-space domain method is compared with TCR [32] and temperature maps estimated after image reconstruction by either conjugate gradient (CG) or SPIRiT [43]. SPIRiT kernels were calibrated using the (fully-sampled) baseline image and reconstructions used a kernel size of 5 and calibration region size of 30, and the SPIRiT regularization parameter was set to equal the median of the absolute value of the k-space data. Given a reconstructed image \( m \) from CG or SPIRiT, temperature phase shifts were estimated as:

\[
\theta_j = \angle(m_j f_j^*), \tag{6}
\]

for each voxel \( j \), where \( f \) is the baseline image with polynomial phase shift estimated by the k-space algorithm.

Temporal resolution simulation

k-Space data of a simulated circular phantom and on-off Gaussian-shaped (FWHM of 0.8 pixels) heat-induced phase shift were generated for a golden angle (GA) radial trajectory at 3 T with 1 receive coil and: FOV = 20 cm; matrix size = 64 x 64; TE = 16 ms; 101 projections for full sampling. The peak phase shift of 1.7 radians corresponded to 13°C heating. The median absolute value of the k-space data was 4.37. Temperature change maps were reconstructed at 4× acceleration (25 projections) using TCR and the k-space method. TCR reconstructions used a weighting factor \( \alpha \) of 100 and 100 iterations. The k-space method used \( \lambda = 10^{-4} \), one baseline image, and a zeroth-order polynomial fit.

Multicoil/Parallel imaging simulation

The same circular object and Gaussian-shaped phase shift were generated with the same settings as the temporal resolution simulation for a single receive coil with uniform sensitivity, and for a simulated 8-channel 3 T receive coil array [44]. k-Space data were synthesized using GA radial
and 2DFT readout trajectories. Temperature maps were reconstructed by the k-space method and by Eq. 6 from images reconstructed by SPIRiT for the 8-channel data. Reconstructions were performed across a range of acceleration factors. The k-space method again used $\lambda = 10^{-4}$, one baseline image, and a zeroth-order polynomial fit.

Phantom heating experiments

To evaluate the model experimentally, a tissue-mimicking gel phantom was sonicated using a Philips Sonalleve MR-HIFU system (Philips Healthcare, Vantaa, Finland) with a 12 mm diameter focus at 100 W and 1.2 MHz for 36 seconds. Gradient echo imaging at 3 T (Philips Achieva, Philips Healthcare, Best, Netherlands) with 2DFT and GA radial readout trajectories and five receive coils were used for temperature imaging. Parameters for the 2DFT and GA radial sequences were: FOV = 40 cm; matrix size = 192 x 192; slice thickness = 7 mm; TR = 32 ms; TE = 16 ms. The multicoil data were compressed to three coils by thresholding the singular values of the data matrix at five percent of the largest singular value. In addition to the k-space domain reconstructions, temperature maps were estimated using Eq. 6 and images reconstructed by CG and SPIRiT. Temperature maps were reconstructed at 96 x 96 resolution with one baseline image, zeroth-order polynomial fit, and $\lambda = 0.003$ (2DFT) and 0.015 (GA radial). A second order finite differencing spatial roughness penalty (with regularization parameter $\beta = 2^{-11}$) was added to the $\theta$ cost function following the method described in section 2G of Ref. [45]. 2DFT data were undersampled by a factor of 5 by retaining every fifth line in either the $x$- or $y$-direction, resulting in 20 lines. GA radial data were undersampled by a factor of 16, resulting in 9 lines.

In vivo model validation

To validate the k-space model in vivo in the absence of heat, sagittal brain images were collected in a healthy volunteer using an 8-channel receive array and GA radial readout trajectory at 3 T (Philips Achieva, Philips Healthcare, Best, Netherlands) under approval of the Institutional Review Board at Vanderbilt University with: FOV = 25.6 cm; matrix size = 128 x 128; slice thickness
= 3 mm; TR = 100 ms; TE = 10 ms. Multicoil data were compressed to four coils using singular value thresholding as in the phantom heating experiments. Temperature maps were estimated using k-space reconstructions and using Eq. 6 with CG- and SPIRiT-reconstructed images. To evaluate temperature errors that could arise in the k-space reconstructions, the algorithm was executed without the \( \theta \) updates (Step 3 was skipped and \( \theta \) was fixed at 0) so that only the baseline image weights and background phase shift were estimated, using seven baseline images and a first order polynomial matrix. Then, keeping those baseline image and polynomial phase estimates fixed by skipping Steps 2 and 4, the algorithm was repeated to fit \( \theta \) with \( \lambda = 0 \). Temperature maps were reconstructed with no acceleration (256 lines; 97 maps), 2\( \times \) acceleration (128 lines; 109 maps), 4\( \times \) acceleration (64 lines; 116 maps), 8\( \times \) acceleration (32 lines; 119 maps), 16\( \times \) acceleration (16 lines; 121 maps), and 32\( \times \) acceleration (8 lines; 121 maps). Errors were averaged over the reconstructed temperature maps to determine the mean errors for each acceleration factor.

**Results**

**Temporal resolution simulation**

The fully-sampled and undersampled k-space trajectories and corresponding magnitude image and complex phase difference temperature map reconstructions of the simulated data are shown in Fig. 1a. Undersampling results in a noisy appearance of the magnitude image, and coherent streaking artifacts in the temperature maps. The temperature in the center voxel is plotted for the true and reconstructed maps in Fig. 1b. The k-space method better tracks the rapid temperature changes achieving finer temporal resolution than TCR. Figure 1c shows temperature maps at 5 seconds. The k-space reconstruction has small truncation errors around the hot spot, but overall much lower error and no visible aliasing artifacts. With TCR, aliasing artifacts could be reduced by increasing temporal regularization, but this would further degrade temporal resolution.
Multicoil/Parallel imaging simulation

Parallel imaging simulation results are shown in Fig. 2. Root-mean-square error within the object, maximum error within the object and excluding the temperature hotspot, and error in the center voxel of the reconstructed temperature maps were calculated for the 2DFT and GA radial reconstructions. The changing interference of coherent aliases at different acceleration factors results in oscillation in the 2DFT SPIRiT errors. Similar patterns are observed to a lesser degree in the single-coil k-space reconstructions at higher acceleration factors. However, the multi-channel k-space reconstructions are free from these artifacts and produce temperature estimates with low error using as few as 4 readout lines. This illustrates that the accuracy of the k-space method’s reconstructions improves with multiple receive channels. GA radial reconstructions using the k-space-based method do not exhibit significant aliasing artifacts in either single- or multi-coil cases. Error remains low for k-space estimates, even when only 2 projection lines are used for temperature reconstruction. In contrast, the SPIRiT reconstructions are increasingly affected by undersampling artifacts as the acceleration factor increases.

Phantom heating experiments

Figure 3 shows experimental results from the 2DFT scan. Image-based reconstructions were highly sensitive to the relative orientation of the coil array (which encoded primarily along $x$) and the undersampled dimension. Aliasing artifacts of the images were reduced when undersampled along the $x$- rather than the $y$-direction. Compared to the CG reconstruction, the SPIRiT algorithm was able to reduce but not fully remove artifacts from undersampling. The k-space temperature reconstructions were free from artifacts, regardless of the undersampled dimension. Temperature maps subsampled along the $x$-direction are evaluated over the timecourse of the heating experiment. Peak heating is overestimated in the SPIRiT temperature maps, whereas temperature changes are underestimated during the heating and cooling segments of the experiment. The undersampled temperature reconstructions using the k-space method are in agreement with the fully-sampled results, both in temperature map appearance and estimated amount of heating.
Results from the GA radial scan are shown in Figure 4. As in the 2DFT case, aliasing artifacts are apparent in CG and SPIRiT reconstructions, but absent in k-space-based reconstructions. When accelerated at $16 \times$, SPIRiT-based temperature estimates are consistently lower than the heating estimated from fully-sampled data. The k-space temperature reconstruction algorithm produces heating estimates that closely match the fully-sampled results throughout the experiment.

In vivo model validation

Figure 5 shows the in vivo model validation results. As expected, the temperature errors increase with acceleration factor for all reconstruction methods. At high acceleration factors, the CG-reconstructed temperature maps have large errors, which are reduced but not removed by using SPIRiT. The k-space method’s maps reflect a globally increased temperature uncertainty resulting from lower SNR, but no significant aliasing artifacts within the brain up until a $32 \times$ acceleration.

Computation time

Table 1 shows the computation time, averaged over 10 repeated reconstructions, for one temperature map from simulated and phantom heating data. Simulated reconstructions did not have the same signal and noise characteristics as the experimentally acquired data, and did not have the additional spatial roughness penalty regularization used in phantom heating reconstructions. The simulation also required more iterations of the algorithm to estimate temperature change, which may contribute to the increased time required.

Discussion

The proposed k-space-based temperature reconstruction approach was demonstrated in simulations and experiments to produce accurate temperature maps at higher levels of acceleration than compared methods. Simulations of a rapid temperature change between $0^\circ C$ and $13^\circ C$ demonstrated the method can reconstruct $4 \times$-undersampled temperature maps with finer temporal resolution than
TCR and with high accuracy. Small errors are observed around the hotspot (RMSE: 0.0047°C; max error: 0.074°C), but undersampling artifacts are not present. TCR maps have more substantial errors, with a streaking pattern typical of radial undersampling (RMSE: 0.23°C; max error: 2.98°C). Although increased temporal regularization could reduce TCR aliasing artifacts, this would decrease the effective temporal resolution.

Simulations using one receive coil and an eight-coil array demonstrated that the k-space algorithm can benefit from multiple coils. For 2DFT and GA radial trajectories, k-space temperature estimates from multi-coil data had lower error within and away from the hotspot using as few as four (2DFT) and two (GA radial) readout lines. Using multiple coils improved the k-space reconstructions for the 2DFT simulation for acceleration factors greater than two. For both 2DFT and GA radial trajectories, k-space temperature maps had lower error than SPIRiT maps for acceleration factors greater than four. These results suggest that using multiple receive coils may be more advantageous for Cartesian trajectories, although additional experiments will be needed to evaluate this. In both Cartesian and non-Cartesian simulations, low-error temperature reconstructions are achieved at higher levels of acceleration using the k-space method than using parallel imaging alone.

A similar pattern is observed in phantom heating experiments. 2DFT and GA radial data were undersampled by factors of 5 and 16, respectively. Results showed that undersampled k-space reconstructions closely match the fully-sampled temperature maps. In both experiments, the k-space reconstructions are free from aliasing artifacts. In contrast, SPIRiT-reconstructed temperature maps suffered from aliasing artifacts and degraded accuracy across the timecourse of the experiments. Unlike the SPIRiT method, we observed that the k-space algorithm was not affected by the undersampling dimension with respect to the direction of coil sensitivity variation, suggesting that the proposed method may be applicable to a wider variety of datasets.

To investigate possible temperature errors that could arise in the method, data from a healthy volunteer were collected without any heating. Above moderate acceleration factors, large errors became apparent in CG temperature reconstructions. SPIRiT maps also had high error at these
factors, although they appeared diminished compared to CG results. Error maps from k-space reconstructions were affected by decreasing SNR, but remained low until reaching an acceleration factor of 32.

Compared to temporal regularization-based approaches such as TCR, the k-space-based algorithm makes no assumptions on the temporal dynamics of temperature, and was shown in simulations to provide superior temporal resolution to TCR. The k-space algorithm also does not require k-space undersampling patterns to change between consecutive acquisitions, as was demonstrated by the 2DFT phantom reconstructions, where the accelerated sampling pattern was held fixed for the entire duration of the experiment. However, higher accelerations may be possible with the proposed method if temporal regularization is added to the objective function in Eq. 2 and an alternating k-space sampling pattern is used.

For the current implementation of the algorithm, the measured compute times for one frame were in the tens of seconds. While not compatible with real-time use, these times are feasible for retrospective reconstructions that would be useful for preclinical studies [46]. Furthermore, the current implementation used only one CPU core, and a multi-threaded or GPU-based implementation of the algorithm may yield compute times compatible with real-time use. As for any reconstruction approach, multi-slice temperature reconstructions could be parallelized across slices, so the in-plane acceleration provided by the k-space algorithm could be leveraged to increase the number of acquired slices without sacrificing temporal resolution.

Conclusions

Accelerated temperature imaging is important to support current and enable future MRI-guided thermal interventions. We have presented an algorithm for temperature reconstruction by fitting a constrained model directly to k-space data. The proposed algorithm is capable of estimating temperature changes from k-space data with high temporal resolution and accuracy, even at high acceleration factors.
Appendix

NLCG algorithm for $\theta$ updates

The steps of the nonlinear conjugate gradient algorithm (J A Fessler, *Image reconstruction: Algorithms and Analysis*, to be published.) used to update the heating-induced phase shift $\theta$ in Step 3 of the algorithm are:

1: repeat

2: Calculate search direction $p$: A column vector $g = \nabla_{\theta} \Psi$ of the first derivatives of the objective function $\Psi(\theta)$ in Eq. 4 with respect to the elements of $\theta$ is first calculated as:

$$
\begin{align*}
    g^n & = \nabla_{\theta} \Psi = \Re \left\{ \text{diag} \left\{ f_j^* e^{-i\theta_j^0} \right\} G^\prime \left( \tilde{y} - G \text{diag} \left\{ e^{i\theta_j^0} \right\} f \right) \right\},
\end{align*}
$$

(7)

where $G$ is a (non-uniform) DFT matrix. Using the Polack-Ribiére conjugate gradient formula, the search direction at iteration $n$ is given by:

$$
\begin{align*}
    p^n & = \begin{cases} -g^n, & n = 0 \\
        -g^n + \frac{\langle g^n - g^{n-1}, g^n \rangle}{\|g^{n-1}\|^2} p^{n-1}, & n \geq 1 \end{cases}
\end{align*}
$$

(8)

3: Calculate step size $\alpha$: $\alpha$ is evaluated subject to non-positivity constraints as follows: If $\theta_j^n > \epsilon$ for some threshold $\epsilon$ ($\epsilon = \pi/1000$ was used in the present work) and $p_j^n > 0$, then set $p_j^n = 0$ since the search direction will lead to violation of the non-positivity constraint at that location, for any $\alpha$. Then solve

$$
\alpha_n = \arg\min_{\alpha} \Psi(\theta^n + \alpha p^n)
$$

(9)

using a backtracking line search. Next, determine if the $\alpha$ returned by the line search causes violation of the non-positivity constraint. Let $z^n = \theta^n + \alpha_n p^n$. If $[z^n]_- \neq z^n$, then set

$$
    p^n = [z^n]_+ - \theta^n
$$

(10)

and perform another backtracking line search with the updated $p^n$:

$$
\alpha_n = \arg\min_{\alpha \in [0,1]} \Psi(\theta^n + \alpha p^n)
$$

(11)
4: Update \( \theta: \theta^{n+1} = \theta^n + \alpha p^n \)

5: until The desired number of iterations is reached.

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Figure captions

Figure 1: Temporal simulation results. (a) 4×-undersampling results in noisy magnitude images and streaking in the temperature maps. (b) The proposed k-space-based reconstruction achieves finer temporal resolution than TCR since it does not depend on temporal regularization to suppress aliasing artifacts. (c) At 5 seconds, aliasing artifacts are visible in the TCR estimate, but not in the k-space estimate. The circle at 5 seconds on the x-axis in (b) indicates the time of the temperature error comparison in (c).

Figure 2: Parallel imaging simulation results. Root-mean-square error, maximum error outside the hotspot, and error in the center voxel are plotted for (a) 2DFT and (b) GA radial trajectories. Black labels along the x-axis indicate acceleration factor, while gray labels indicate the corresponding number of readout lines used in the reconstruction. k-Space reconstruction errors are generally lower than SPIRiT errors. Multi-coil k-space estimates have lower error than single-coil estimates, indicating that the method benefits from parallel imaging.

Figure 3: 2DFT experiment results. (a) Magnitude images reconstructed with full sampling (left) and 5× undersampling along x- and y-directions. (b) Temperature maps reconstructed at peak heat with x- and y-subsampling schemes using CG, SPIRiT, and k-space-based models. (c) Temperature maps reconstructed along the timecourse of the heating experiment with no undersampling using CG (top) and undersampling along the x-direction using SPIRiT-reconstructed images (middle) and k-space data (bottom). Plot of temperature evolution in the central voxel as a function of time for fully-sampled (CG) and 5×-undersampled (SPIRiT and k-space-domain) temperature maps. Circles along the x-axis indicate times of the displayed temperature maps.
Figure 4: Golden angle radial results. (a) Magnitude images reconstructed with full sampling (left) and $16 \times$ undersampling with CG (middle) and SPIRiT reconstruction (right). (b) Temperature maps reconstructed at peak heat using fully-sampled CG and $16 \times$-undersampled CG, SPIRiT, and k-space-based models. (c) Temperature maps reconstructed along the timecourse of the heating experiment with no undersampling using CG (top) and $16 \times$-undersampling using SPIRiT-reconstructed images (middle) and k-space data (bottom). Plot of temperature evolution in the central voxel as a function of time for fully-sampled (CG) and $16 \times$-undersampled (SPIRiT and k-space-domain) temperature maps. Circles along the $x$-axis indicate times of the displayed temperature maps.

Figure 5: In vivo model validation results. Magnitude images reconstructed at different acceleration factors (top). Comparison of mean positive temperature errors over $0.25^\circ$C for CG, SPIRiT, and k-space temperature reconstructions. Moderate acceleration leads to large temperature errors in the CG and SPIRiT reconstructions, but the k-space reconstructions suffer only an apparent loss in SNR in the brain up to a high acceleration factor of $32 \times$.

Table 1: Computation Time and Iteration Count

<table>
<thead>
<tr>
<th>Dataset, acceleration factor</th>
<th>Time (s)</th>
<th>Number of iterations</th>
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</thead>
<tbody>
<tr>
<td>Simulated phantom, $4 \times$</td>
<td>74.11</td>
<td>Steps 1-5: 6; Step 6: 3</td>
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<tr>
<td>2DFT gel phantom, $5 \times$</td>
<td>12.88</td>
<td>Steps 1-5: 3; Step 6: 3</td>
</tr>
<tr>
<td>GA radial gel phantom, $16 \times$</td>
<td>11.53</td>
<td>Steps 1-5: 3; Step 6: 3</td>
</tr>
</tbody>
</table>
Temporal simulation results. (a) 4x-undersampling results in noisy magnitude images and streaking in the temperature maps. (b) The proposed k-space-based reconstruction achieves finer temporal resolution than TCR since it does not depend on temporal regularization to suppress aliasing artifacts. (c) At 5 seconds, aliasing artifacts are visible in the TCR estimate, but not in the k-space estimate. The circle at 5 seconds on the x-axis in (b) indicates the time of the temperature error comparison in (c).
Parallel imaging simulation results. Root-mean-square error, maximum error outside the hotspot, and error in the center voxel are plotted for (a) 2DFT and (b) GA radial trajectories. Black labels along the x-axis indicate acceleration factor, while gray labels indicate the corresponding number of readout lines used in the reconstruction. k-Space reconstruction errors are generally lower than SPIRiT errors. Multi-coil k-space estimates have lower error than single-coil estimates, indicating that the method benefits from parallel imaging.

114x93mm (300 x 300 DPI)
2DFT experiment results. (a) Magnitude images reconstructed with full sampling (left) and 5x undersampling along x- and y-directions. (b) Temperature maps reconstructed at peak heat with x- and y-subsampling schemes using CG, SPIRiT, and k-space-based models. (c) Temperature maps reconstructed along the timecourse of the heating experiment with no undersampling using CG (top) and undersampling along the x-direction using SPIRiT-reconstructed images (middle) and k-space data (bottom). Plot of temperature evolution in the central voxel as a function of time for fully-sampled (CG) and 5x-undersampled (SPIRiT and k-space-domain) temperature maps. Circles along the x-axis indicate times of the displayed temperature maps.

349x597mm (300 x 300 DPI)
Golden angle radial results. (a) Magnitude images reconstructed with full sampling (left) and 16x undersampling with CG (middle) and SPIRiT reconstruction (right). (b) Temperature maps reconstructed at peak heat using fully-sampled CG and 16x-undersampled CG, SPIRiT, and k-space-based models. (c) Temperature maps reconstructed along the timecourse of the heating experiment with no undersampling using CG (top) and 16x-undersampling using SPIRiT-reconstructed images (middle) and k-space data (bottom). Plot of temperature evolution in the central voxel as a function of time for fully-sampled (CG) and 16x-undersampled (SPIRiT and k-space-domain) temperature maps. Circles along the x-axis indicate times of the displayed temperature maps.
In vivo model validation results. Magnitude images reconstructed at different acceleration factors (top). Comparison of mean positive temperature errors over 0.25°C for CG, SPIRiT, and k-space temperature reconstructions. Moderate acceleration leads to large temperature errors in the CG and SPIRiT reconstructions, but the k-space reconstructions suffer only an apparent loss in SNR in the brain up to a high acceleration factor of 32.

157x91mm (300 x 300 DPI)