|$B_1^+|$-Selective Excitation Pulse Design Using the Shinnar–Le Roux Algorithm

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Abstract

A new mathematical treatment and algorithm for the design of |$B_1^+$|-selective RF excitation pulses is presented and validated. The algorithm is based on a rotated Shinnar-Le Roux pulse design algorithm, wherein the pulse’s frequency modulation waveform is directly designed by the algorithm, and its amplitude modulation waveform takes the place of the gradient field. A new pulse configuration is described that enables excitation of large tip-angle slice-selective profiles. Experiments were performed to validate the pulses, and simulations were performed to characterize the pulses’ sensitivity to off-resonance, and to compare them to adiabatic (BIR-4) pulses.

Keywords: RF pulse design, Selective Excitation, Shinnar-Le Roux algorithm, Rotating Frame Selective Excitation

1. Introduction

In 1980 Hoult described a class of ‘Rotating Frame Selective Excitation Pulses’ that selectively excite magnetization based on the strength of the RF transmit field (|$B_1^+$|) they experience [1]. The pulses were intended for use in rotating frame imaging [2, 3, 4], but could be used for slice selection in any of several RF gradient-based imaging methods that have been proposed since [5, 6, 7, 8]. The pulses were based on the assumption of a large and constant $B_{1,x}$ gradient field. When this field was switched on, initially-longitudinal magnetization would precess around it in the $y$-$z$ plane. Hoult showed that by modulating the $B_{1,y}$ field, magnetization could be selectively excited based on the...
magnitude of the $B_{1,x}$ component (the strength of $B_{1,y}$ was implicitly assumed to be constant across space). The pulses were designed by analogy to $B_0$-selective excitation, wherein $B_{1,x}$ was treated as the longitudinal gradient field, and $B_{1,y}$ the perpendicular field responsible for flipping magnetization. Assuming a constant $B_{1,x}$ waveform was played (analogous to the constant $B_0$ gradient used in conventional slice-selective excitation), pulse design then amounted to designing the $B_{1,y}$ modulation required to obtain the desired slice profile. Somewhat improved design methods and results were described several years later by Karczmar [9] and Hedges [10]. However, those reports did not provide a clear prescription to design the pulses to meet target slice characteristics, analogous to the Shinnar–Le Roux algorithm, and the theory underlying the pulses’ function and performance remained largely undescribed.

We recast the $|B_{1}^+|$-selective pulse design problem as one of designing a frequency modulation waveform rather than a $B_{1,y}$ field component, and show that the small-tip-angle Shinnar–Le Roux (SLR) algorithm [11, 12, 13, 14, 15, 16] can be used to directly design this waveform for excitations ($0–90^\circ$ tip angles) and inversions. The result is a simple and fast pulse design approach that inherits the ease-of-use of SLR, provides a substantial improvement in the selectivity of the pulses over previous design methods, and enables the excitation of larger tip-angles. The following sections formulate the pulse design method, present simulation results that characterize the pulses’ off-resonance sensitivity and compare them to adiabatic pulses, and present experimental results that validate the pulses’ function. Preliminary aspects of this work were presented in Ref. [17].

2. Theory

Rotated coordinate system

The proposed algorithm directly designs a frequency modulation waveform $\Delta \omega(t)$ that is paired with a constant amplitude modulation waveform $\omega_1(t)$. $\Delta \omega(t)$ takes the place of the transverse RF field waveform that is designed by the SLR algorithm, and $\omega_1(t)$ takes the place of the accompanying gradient waveform for slice selection, and is scaled by the magnitude of the RF field ($|B_{1}^+|$), rather than by a spatial coordinate. This configuration is achieved by rotating the logical axes in the pulse design problem, as shown in Fig. 1, so that the $\Delta \omega$ field component lies along the $-x$-axis, and the $\omega_1$ component lies along the $z$-axis.
Figure 1: The SLR algorithm designs pulses with field components in a plane perpendicular to an encoding field. Conventionally, the encoding field is the $B_0$-directed gradient field, and the algorithm designs the in-phase and quadrature components of the RF field. To design a $|B_1^+|$-selective excitation pulse using SLR, the coordinate axes of Bloch equation can be rotated so that the RF amplitude field ($\omega_1$, the encoding field) lies along the $z$-axis, and the frequency modulation field ($\Delta\omega$) lies along the $x$-axis, and becomes the target of pulse design.

**Target excitation profile**

The SLR algorithm is based on relating target magnetization profiles ($M_x$, $M_y$, and $M_z$) to spinor parameter profiles ($\alpha$ and $\beta$) whose discrete Fourier transform (DFT) coefficients can be inverted to obtain the RF pulse that produces them. To apply the algorithm to design a $\Delta\omega(t)$ waveform that excites a slice along the $|B_1^+|$ axis, we must express target excitation profiles in terms of $\alpha$ and $\beta$ in the rotated coordinate system. The inverse SLR transform can then compute the $\Delta\omega(t)$ waveform that corresponds to those parameters. Given initial magnetization $M_{zy}^-=M_z^- + iM_y^-$, and $M_x^-$, the magnetization after a pulse with parameters ($\alpha, \beta$) in the rotated coordinate system will be:

$$
\begin{pmatrix}
M_{zy}^+
M_{zy}^{+*}
M_x^+
\end{pmatrix}
= \begin{pmatrix}
(\alpha^*)^2 & -\beta^2 & 2\alpha^*\beta \\
-(\beta^*)^2 & \alpha^2 & 2\beta^*
\end{pmatrix}
\begin{pmatrix}
M_{zy}^-
M_{zy}^{*-}
M_x^-
\end{pmatrix}. 
$$
For initial magnetization at thermal equilibrium \((M_x^-, M_y^-, M_z^-) = (0, 0, 1))\), the excited transverse magnetization will be:

\[
M_x^+ = 2 \Re\{\alpha^* \beta^*\} = 2(\alpha_R \beta_R - \alpha_I \beta_I) \\
M_y^+ = 3 \{(\alpha^*)^2 - \beta^2\} = -2(\alpha_R \alpha_I + \beta_R \beta_I),
\]

where the \(R\) and \(I\) subscripts denote the real and imaginary parts of the parameters, respectively.

As in conventional linear-phase SLR pulse design and previous \(|B_1^+|\)-selective design methods, we will design pulses that produce constant- (specifically, zero-) phase profiles across the excited slice after a refocusing lobe is applied, so that \(M_y^+ = 0\). For these pulses \(\beta_I\) will also be zero after refocusing. If we further restrict our consideration to small-tip-angle pulses with zero gradient area [18], then \(\alpha_R \approx 1\) and \(\alpha_I \approx 0\) [18]. In this case,

\[
M_x^+ = 2\beta_R,
\]

and \(M_y^+ = 0\). Therefore, \(\beta_R\) is the parameter of interest for digital filter design in the \(|B_1^+|\)-selective SLR algorithm. Conveniently, because \(\beta_R\) lies along the same axis in both the rotated and conventional coordinate systems, and because we have that \(M_x^+ = -2\beta_R\) for a conventional small-tip-angle slice-selective pulse with zero gradient area [18], the same ripple relationships provided in Ref. [16] also apply to \(|B_1^+|\)-selective pulse design.

Figure 2 illustrates the target \(\beta\) profile configuration. Unlike conventional slice-selective excitation, a \(|B_1^+|\)-selective slice profile cannot be centered at \(|B_1^+| = 0\), since excitation cannot occur with zero RF field. Thus, the slice profile must be shifted away from this point. A slice-selective excitation is conventionally shifted using frequency modulation of the RF pulse; however, this would result in complex \(\beta\) DFT coefficients, and subsequently a complex-valued \(\Delta\omega(t)\) waveform. The \(\Delta\omega(t)\) waveform must be real-valued to be physically realizable, which dictates that the \(\beta\) DFT coefficients must be purely imaginary, since a small-tip RF pulse designed by SLR is \(\pi/2\) out of phase with its \(\beta\) DFT coefficients [16]. The required purely imaginary \(\beta\) DFT coefficients can be obtained by specifying an odd and dual-band (anti-symmetric) \(\beta\) profile [19]. Thus, the target \(\beta\) profile must be real-valued, dual-band, odd, and zero at \(|B_1^+| = 0\).

**\(\beta\) filter design**

A real-valued, odd, and dual-band \(\beta\) profile and its corresponding DFT coefficients can be designed in several ways. For well-separated passbands (i.e., centered at sufficiently high \(|B_1^+|\)),
the process can start with a conventional single-band linear-phase finite impulse response filter designed using a weighted-least squares method. That filter is then duplicated, and the duplicates are frequency modulated to opposite center frequencies and subtracted from each other. This is equivalent to modulation of the single-band filter by a sine function at the center frequency. For very close passbands (i.e., passbands close to $|B_1^+| = 0$) however, ripples from one passband can distort the other. In these cases, an odd, dual-band $\beta$ filter can be designed directly using weighted-least-squares. The distortions could also be mitigated using a phase-correction method [20]. Once the $\beta$ filter is designed, assuming small excitation angles the inverse SLR transform reduces to a simple scaling of the filter coefficients to obtain the $\Delta\omega(t)$ waveform.
Figure 3: Amplitude modulation waveform ($\omega_1(t)$) construction. (Left Column) With no pre- or rewinding, energy is only deposited at positive frequencies, leading to nonzero $\alpha_I$ and $\beta_I$ profiles. (Middle Column) By adding a rewinder to the waveform, energy will be deposited at both positive and negative frequencies, leading to zero $\beta_I$ but nonzero $\alpha_I$. (Right Column) Adding a prewinder to the waveform yields $\alpha_I = 0$ as required.

**Amplitude modulation waveform construction**

The SLR algorithm conventionally designs an RF pulse that accompanies a constant gradient waveform. In $|B_1^+|$-selective pulse design, the amplitude modulation RF waveform $\omega_1(t)$ replaces the gradient waveform. In the small-excitation angle regime, the $\alpha$ profile at the end of a pulse with duration $T$ is [18]:

$$\alpha(|B_1^+|) = e^{-\frac{i\pi}{2}|B_1^+|\int_0^T \omega_1(t)dt},$$

and the $\beta$ profile is:

$$\beta(|B_1^+|) = \frac{i}{2} e^{\frac{\pi i}{2}|B_1^+|\int_0^T \omega_1(t)dt} \int_0^T \Delta\omega(t)e^{-i\pi|B_1^+|\int_0^T \omega_1(s)ds} dt.$$

From the Fourier integral term in $\beta$, it is evident that if $\omega_1(t)$ is constant and comprises no pre- or rewinder lobes before or after the $\Delta\omega(t)$ waveform, then $\Delta\omega(t)$ will only deposit energy at positive frequencies, as depicted in the left column of Fig. 3. A real and odd $\beta$ profile can only be produced if energy is deposited symmetrically as a function of frequency, and therefore cannot be produced with this trajectory. Further, since the area under the $\omega_1(t)$ waveform is non-zero, $\alpha_I \neq 0$, which is unacceptable. The pulse’s trajectory can be shifted to be symmetric by adding a rewinding lobe with half the main lobe’s area to the end of the pulse, as shown in the middle column of Fig. 3. However, since the area under the waveform will still be nonzero, $\alpha$ will still have an imaginary component. Placing a prewinding lobe at the beginning of the pulse achieves a zero area, so that $\alpha = 1$ as required.
Figure 4: Maintaining selectivity at large tip-angles. (a) At small tip-angles such as 10°, $\alpha_I \approx 0$ as assumed in the design. As the tip angle is increased to 90°, $\alpha_I$ increases dramatically, and degrades selectivity of the pulse outside the passband. By splitting the 90° pulse into its two halves, reflecting those halves and playing them out during the pre- and rewinding $\omega_1(t)$ lobes, the selectivity of the pulse is restored. (b) The split-and-reflect strategy also enables selective inversions.

**Maintaining selectivity at large tip-angles**

Figure 4a shows that as a $|B_1^+|$-selective pulse is scaled to excite a large tip-angle, $\alpha_I$ grows and degrades the excited profile by creating a large unwanted $M_y$ component (Eq. 2), particularly in the stopband. This is a consequence of the Hilbert transform relationship between the amplitude and phase of $\alpha$ for a minimum-power pulse [16]. To mitigate this error and achieve accurate large-tip-angle excitations and inversions, the two halves of the $\Delta \omega(t)$ waveform can be reflected and played out on the pre- and rewinding $\omega_1(t)$ lobes. The amplitude of the whole pulse is also divided by two.
to achieve the target tip angle. With this modification, the size of the $\alpha_I$ component is reduced dramatically, and selectivity is restored at large tip-angles. This works because, for each half of the pulse, time-reversal flips the phase of $\alpha$, and the division by two increases its amplitude, which reduces its phase by the Hilbert transform relationship. This leads to a combined $\alpha$ parameter for each half that is dominated by $|\alpha|^2$ when the pulse halves are played back-to-back with their time-reversed copies. Figure 4b shows that with this modification, selective inversions can be designed.

**Overall algorithm**

Given the RF digital-to-analog conversion dwell time $\Delta_t$, the time-bandwidth product $TB$, the pulse type (small-tip, excitation, inversion, or saturation), the tip angle $\theta$, the passband width $PBW$ (Gauss), the passband center $PBC$, and the passband ($\delta_1, e$) and stopband ($\delta_2, e$) ripple levels (units of $M_0^{-1}$), the steps of the proposed $|B_4^+|$-selective pulse design algorithm are:

1. Calculate the half-pulse duration $T$ and the number of samples in the half-pulse $n$:

   $$T = \frac{TB}{\gamma (PBW)}$$

   $$n = 2 \left\lceil \frac{T}{2\Delta_t} \right\rceil$$

   where $\gamma$ is the gyromagnetic ratio in radians per second per Gauss.

2. Calculate the inputs for finite impulse response (FIR) $\beta$ filter design. The required inputs to the commonly-used MATLAB (Mathworks, Natick, MA, USA) `firls` function are:

   $n$: The number of samples in the filter.

   $$f = \left[ 0 \ (1-w)\frac{TB}{n} \ (1+w)\frac{TB}{n} \ 1 \right] : \text{Normalized band edges.}$$

   $$m = \left[ 1 \ 1 \ 0 \ 0 \right] : \text{Weights at band edges.}$$

   $$wts = \left[ 1 \ \frac{\delta_1}{\delta_2} \right] : \text{Band error weights.}$$

   Here, $w$ is the fractional transition width, which is:

   $$w = \frac{d_\infty(\delta_1, \delta_2)}{TB}$$

   where the $\beta$ ripples ($\delta_1, \delta_2$) are calculated from ($\delta_{1,e}, \delta_{2,e}$) and the pulse type using the relationships and $d_\infty$ function defined in Ref. [16].

3. Run the FIR filter design tool to design the $\beta$ filter, producing length-$n$ coefficient vector $h$. 

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4. Sine-modulate the $\beta$ filter coefficients to the desired passband center:

$$h_j \leftarrow 2h_j \sin \left( \gamma \text{PBC} \left( j\Delta_t - \frac{T}{2} \right) \right), \quad j = 0, \ldots, n - 1.$$  

5. Split and reflect the modulated filter, producing length-$2n$ coefficient vector $\tilde{h}$:

$$\tilde{h}_j = \begin{cases} 
    h_{n/2-1-j}/2, & j = 0, \ldots, \frac{n}{2} - 1, \\
    h_{j-n/2}/2, & j = \frac{n}{2}, \ldots, \frac{3n}{2} - 1, \\
    h_{n-1-(j-3n/2)/2}, & j = \frac{3n}{2}, \ldots, 2n - 1. 
\end{cases}$$  

6. Scale to the correct flip angle and convert to Hz to get the full, sampled $\Delta\omega(t)$ waveform $\Delta\omega$:

$$\Delta\omega,j = \frac{\theta}{2\pi\Delta_t} \tilde{h}_j, \quad j = 0, \ldots, 2n - 1.$$  

7. Build the sampled, normalized $\omega_1(t)$ waveform $\omega_1$:

$$\omega_1,j = \begin{cases} 
    -1, & j = 0, \ldots, \frac{n}{2} - 1, \\
    1, & j = \frac{n}{2}, \ldots, \frac{3n}{2} - 1, \\
    -1, & j = \frac{3n}{2}, \ldots, 2n - 1. 
\end{cases}$$  

3. Methods

3.1. Experiments

Phantom experiments were performed to validate control of flip angle, time-bandwidth product, and centering of the pulses. $|B^+_1|$-Selective pulses were designed in MATLAB$^1$ and deployed on a 31 cm 4.7 T Varian spectrometer (Agilent, Santa Clara, CA, USA) with a 38 mm Litz volume coil (Doty Scientific, Columbia, SC, USA) for transmit and receive and a phantom containing a CuSO$_4$ solution with $T_1 \approx 200$ ms. The pulses were used for excitation in a 3D gradient-recalled echo sequence with FOV $30 \times 30 \times 100$ mm, $32 \times 32 \times 32$ matrix size, 50 or 100 ms TR and 5 ms TE as measured from the center of the pulses. The pulses were sampled with a 4 $\mu$s dwell time, and frequency modulation was converted to phase modulation. To account for finite RF amplifier rise times, 40-sample ramps were placed on either end of the $\omega_1(t)$ waveforms, which were immediately followed by 20-sample rewinders to cancel the area of the ramps. These ramps and rewinders are

$^1$MATLAB code to design $|B^+_1|$-selective pulses, and to generate most of the figures in this article is available at http://www.vuiis.vanderbilt.edu/~grissowa/
visible on the waveforms in Fig. 5(a,c). To measure the $|B^+_1|$-selective profile of each pulse, the 3D scans were repeated over a range of nominal flip angles, each of which corresponded to a single point in the $|B^+_1|$ measurement range. Two additional scans were collected to calculate an off-resonance field map, which was used to mask out voxels from the $|B^+_1|$-selective excitation images that were far from resonance. Those scans had the same volume coverage and matrix size, and a 50 ms TR, but used a 30° Gaussian excitation and TEs of 5 and 6 ms, so that a field map could be calculated from their phase difference [21]. Then, from each $|B^+_1|$-selective excitation pulse’s set of 3D acquisitions, the signal for each $|B^+_1|$ was calculated from the corresponding images as the magnitude of the complex average of signal from voxels with off-resonance within ±5 Hz, and inside an object mask derived by thresholding one of the off-resonance map acquisition images at 15% of the peak image magnitude.

3.2. Simulations

Simulations were performed to characterize the sensitivity of $|B^+_1|$-selective pulses to off-resonance, and to compare them to BIR-4 adiabatic pulses [22] in terms of off-resonance sensitivity and threshold $|B^+_1|$. A hard pulse approximation-based Bloch simulator was used [16], with a 2 µs dwell time for the off-resonance simulation, and a 4 µs dwell time for the BIR-4 comparison. The simulations assumed excitation of $^1$H, so that $\frac{\gamma}{2\pi} = 4257$ Hz/Gauss.

4. Results

4.1. Experiments

Figure 5 shows the pulses played out in the experiments and the resulting measured $|B^+_1|$-selective profiles. Figure 5a shows a comparison of signal profiles for nominal 15° and 30° excitations, with duration 2.83 ms, 0.4 Gauss slice width, TB = 2, and centered at 0.4 Gauss. The signal intensity from the 30° excitation is larger and consistent with increased excitation and $T_1$-weighting. Figure 5b shows a comparison of TB = 2 (2.83 ms) and TB = 6 (7.54 ms) pulses and signal profiles, with a nominal 15° flip angle, 0.4 Gauss slice width, and centered at 0.4 Gauss. Figure 5c shows a comparison of the 15° TB = 2 excitations, centered at 0.2 and 0.4 Gauss. The two pulses’ profiles are centered in the intended locations, but otherwise appear very similar. All profiles are more accurate for low $|B^+_1|$, likely due to RF amplifier droop across the pulse durations at high $|B^+_1|$. The TB = 6 pulse’s profile appears to be most affected by droop, due to its much longer duration.
4.2. Simulations

Figure 6 shows the off-resonance simulation results. Four $|B_1|_\perp$-selective pulses were simulated: two 3.1 ms TB = 2 pulses at 30° and 90°, centered at 2 and 4 Gauss, with 0.3 Gauss passband width, and two 12.5 ms TB = 8 pulses, for the same flip angles, profile centering and passband widths. All four designs used $\delta_1,e = \delta_2,e = 0.01$. The two-dimensional patterns of unwanted excitation due to off-resonance appear the same for a given duration. This suggests that off-resonance sensitivity primarily depends on pulse duration and the shape of the $\omega_1(t)$ waveform, rather than on the flip angle and profile centering, which are characteristics that determine the shape and amplitude of the $\Delta \omega(t)$ waveform. As might be expected, near $|B_1|_\perp = 0$, the shorter 3.1 ms pulse appears to have a wider frequency bandwidth over which unwanted excitation is insignificant. Further, in all cases the unwanted excitation decays rapidly as $|B_1|_\perp$ increases. Note that the $M_{xy}$ patterns shown in Fig. 6 are Hermitian symmetric about the $|B_1|_\perp$ axis, and are therefore displayed only for positive off-resonance frequencies.
Figure 6: Off-resonance simulation. The top row shows $|M_{xy}|$ patterns excited by 3.1 ms TB = 2 pulses at 30° and 90°, centered at 2 and 4 Gauss, with 0.3 Gauss passband width. The bottom row shows 12.5 ms TB = 8 pulses, for the same flip angles, profile centering and passband widths. The pattern of unintended excitation appears to depend primarily on the duration of the pulse, rather than the amplitude or centering of the pulses. There is larger unintended excitation for lower $|B_1^+|$.

Figure 7 shows the BIR-4 comparison results. A 4.7 ms, TB = 4 $|B_1^+|$-selective pulse was designed to excite a 45° tip angle, with a passband width of 0.4 Gauss, and ripples $\delta_{1,e} = 0.01$ and $\delta_{2,e} = 0.4$. The high $\delta_{2,e}$ was used to reflect the fact that the stopband above the passband was a ‘don’t-care’ region. The passband was placed as close to $|B_1^+| = 0$ as possible, so direct weighted-least squares dual-band FIR filter design was used to design the $\beta$ filter. Two BIR-4 pulses were then designed: one with the same 4.7 ms duration as the $|B_1^+|$-selective pulse, and one longer 5.9 ms pulse. The 4.7 ms BIR-4 pulse design used $\Delta \omega^0 = 100\pi/T$ radians/second, $\beta = 10$, and $\kappa = \tan^{-1} 20$ [22]. These parameters were empirically selected to match the threshold $|B_1^+|$ and passband ripple of the $|B_1^+|$-selective pulse. The 5.9 ms BIR-4 pulse design used the same $\Delta \omega^0$ and $\beta$, but its longer duration enabled use of a less-aggressive $\kappa = \tan^{-1} 15$. All pulses are plotted in Fig. 7a. Note that there is a $\pi + \pi/8$ phase shift between the central and outer lobes of the BIR-4
pulses’ $\omega_1(t)$ waveforms, to affect the 45° tip angle. Figure 7b plots the $|M_{xy}|$ profile of each pulse at 0 Hz. All three pulses have approximately the same threshold $|B_1^+|$, and approximately the same ripple across the passband. The longer 5.9 ms BIR-4 pulse achieved the same threshold $|B_1^+|$ as the 4.7 ms BIR-4 pulse, without requiring a large $\kappa$. Figure 7c compares the off-resonance sensitivity of the three pulses. The pulses all have similar off-resonance sensitivity near $|B_1^+| = 0$, in the transition up to their passbands. In the passband, the $|B_1^+|$-selective pulse appears to have similar off-resonance sensitivity to the 4.7 ms BIR-4 pulse, but the 5.9 ms BIR-4 pulse is significantly more robust to off-resonance than either 4.7 ms pulse.

![Figure 7: BIR-4 comparison](image)

(a) Comparison of 45° RF waveforms, showing the 4.7 ms TB = 4 $|B_1^+|$-selective excitation pulse, and two BIR-4 pulses: a duration-matched 4.7 ms pulse with parameters tuned for minimum threshold $|B_1^+|$, and a longer 5.9 ms pulse with the same transition width but better robustness to off-resonance. (b) Simulated $|M_{xy}|$ profiles at 0 Hz, illustrating that the three pulses have approximately the same threshold $|B_1^+|$ at which a 45° excitation is achieved. (c) Simulated $|M_{xy}|$ patterns over a 1 kHz bandwidth. The three pulses have similar off-resonance sensitivity for low $|B_1^+|$. As $|B_1^+|$ increases, the 4.7 ms $|B_1^+|$-selective and BIR-4 pulses have similar distortion patterns, but the 5.9 ms BIR-4 pulse has a more uniform excitation pattern across $|B_1^+|$ and off-resonance.

5. Discussion

The proposed algorithm extends the attractive properties of the Shinnar-Le Roux algorithm to the design of $|B_1^+|$-selective pulses. These include speed and the ability to predict a slice profile
analytically, and to thereby make tradeoffs between pulse parameters before ever designing a pulse and evaluating it. This eliminates the need for a guess-and-check approach to pulse design and makes the design process more accessible to non-experts. Further, previous methods for $|B_{1+}^+|$-selective pulse design focused on the design of the $y$-component of the RF field, and assumed that the amplitude of the overall field was independent of that component [9, 10]. By directly designing the frequency modulation waveform rather than the $y$-component of the RF field, the proposed algorithm eliminates a source of approximation error in the pulse design that may have hampered previous methods.

The development of the algorithm was based on an assumption of small excitation angles. While the described split-and-reflect modification to the pulses enables pulses designed by the algorithm to excite selective large-tip-angle profiles up to $180^\circ$ inversions, there will still be some loss in selectivity due to the bandwidth narrowing effect [23]. Attaining the most accurate large-tip excitations will require the development of a novel approach to inverting the $\beta$ profile along a bipolar trajectory, subject to a zero-phase $\alpha$. Recent advances in multidimensional SLR pulse design may lead to the development of such a method in the future [24, 25]. The design of $|B_{1+}^+|$-selective refocusing pulses remains an open problem and will require a different problem formulation than that developed here. Previous reports of $|B_{1+}^+|$-selective pulse design approaches [9, 10] did not address the design of pulses with tip angles greater than $90^\circ$.

In addition to use in RF-encoded imaging, $|B_{1+}^+|$-selective pulses designed by the proposed algorithm may replace adiabatic pulses in some applications, especially when broad robustness to off-resonance is not a primary design objective. The ability to design $|B_{1+}^+|$-selective pulses using the SLR algorithm makes them more intuitive to use than adiabatic pulses, which typically depend on multiple, interdependent and unintuitive parameters that must be empirically tuned to obtain a useful pulse. The proposed algorithm will enable the user to directly specify the range of $|B_{1+}^+|$ over which a given tip angle is desired, within a given tip angle error tolerance, and the algorithm will produce the shortest possible pulse that meets those requirements. Future work may also reveal a connection between $|B_{1+}^+|$-selective and adiabatic pulses, that could enable the straightforward design of adiabatic pulses using the SLR algorithm.
6. Conclusion

A new mathematical treatment and algorithm for $|B_1^+|$-selective RF pulse design was introduced and characterized in simulations and experiments. It is based on the direct design of a frequency modulation waveform using a rotated Shinnar-Le Roux slice-selective pulse design algorithm, and it inherits the desirable properties of the Shinnar-Le Roux algorithm. It can design accurate small-tip, 90° excitation/saturation, and inversion pulses, and in some applications it may be a compelling alternative to adiabatic pulses due to its ease-of-use.

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